

RADIATIVELY INDUCED LORENTZ AND CPT VIOLATION IN QED AT FINITE TEMPERATURE

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In this paper we evaluate the induced Lorentz and CPT violating Chern-Simons term in the QED action at finite temperature. We do this using the method of derivative expansion of fermion determinants. Also, we use the imaginary-time formalism to calculate the temperature dependence of the Chern-Simons term.

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Recently, the problem of the radiatively induced Chern-Simons term in 3+1 dimensions in quantum field theory has been addressed in several of papers. Particularly, this term has been of much interest in study of models that violate Lorentz and CPT symmetry [1–4]. In Quantum Electrodynamics, it is known that the Lorentz and CPT symmetry is destroyed by adding the Chern-Simons term to the Lagrangian density, $\mathcal{L}_{CS} = \frac{1}{2}k_\mu\epsilon^{\mu\alpha\beta\gamma}F_{\alpha\beta}A_\gamma$ [3,5,6], where k_μ is a constant 4-vector. The Chern-Simons term can be induced by CPT and Lorentz violating axial quantity, $b^\mu\gamma_\mu\gamma_5$, in the Lagrangian for massive fermions. Therefore, we get the modified quantum electrodynamics which predicts birefringence of light in vacuum and observation of distant galaxies puts a stringent bound on k_μ [5–8].

The purpose of this paper is to compute the induced Chern-Simons term and analyze the behavior of the coefficient of this term when we are taking temperature into account. To do this, we use derivative expansion method of the fermion determinant [9] and the imaginary-time formalism developed by Matsubara.

Let us consider a modified QED theory described by Lagrangian density

$$\mathcal{L} = \bar{\psi}[i\partial - m - \gamma_5\mathbf{b} - e\mathbf{A}]\psi, \quad (1)$$

where b_μ is a constant 4-vector coupled to axial current and A_μ is the external field. We are using natural units, $c = \hbar = 1$, and the Dirac representation for Dirac matrices.

The corresponding generating functional is

$$Z[A] = \int D\bar{\psi}(x)D\psi(x) \exp \left[i \int \mathcal{L} d^4x \right] \quad (2)$$

We substitute Eq.(1) into Eq.(2) and integrate over the fermion field we obtain

$$Z[A] = \text{Det}(i\partial - m - \gamma_5\mathbf{b} - e\mathbf{A}) = \exp[iS_{eff}[A]]. \quad (3)$$

No Legendre transformation is required to go from the connected vacuum functional to the effective action. Thus, The effective action can be written as

$$S_{eff}[A] = -iTr \ln[i\partial - m - \gamma_5\mathbf{b} - e\mathbf{A}]. \quad (4)$$

Let us write the trace in Eq.(4) in the following equivalent form,

$$Tr \ln[i\partial - m - \gamma_5\mathbf{b} - e\mathbf{A}] = Tr \ln[i\partial - m - \gamma_5\mathbf{b}] + F[A] \quad (5)$$

where

$$F[A] = \int_0^1 dz Tr \left[\frac{1}{i\partial - m - \gamma_5\mathbf{b} - ze\mathbf{A}(x)} e\mathbf{A}(x) \right]. \quad (6)$$

As ∂_μ and $A_\mu(x)$ do not commute, and to perform the momentum space integration of the second term in Eq.(5), we use the notation $i\partial \rightarrow \not{p}$ and $\mathbf{A}(x) \rightarrow \mathbf{A}(x - i\frac{\partial}{\partial p_\mu})$. Then the Eq.(4) can be written as

$$S_{eff}[A] = S_{eff}^{(0)}[A] + S_{effe}^{(1)}[A], \quad (7)$$

where

$$S_{eff}^{(0)}[A] = -iTr \ln[i\partial - m - \gamma_5\mathbf{b}] \quad (8)$$

and

$$S_{eff}^{(1)}[A] = i \int_0^1 dz \int \frac{d^4p}{(2\pi)^4} \times \text{tr} \left[\frac{1}{\not{p} - m - \gamma_5\mathbf{b} - ze\mathbf{A}(x - i\frac{\partial}{\partial p_\mu})} \mathbf{A}(x) \right]. \quad (9)$$

The Eq.(8) has been analyzed in detail in Ref. [6]. Here, we study the second term of $S_{eff}^{(1)}[A]$, and we are keeping only first order derivative terms which are linear in \mathbf{b} and quadratic in \mathbf{A} . Using the operator expansion

$$\frac{1}{A - B} = \frac{1}{A} + \frac{1}{A}B\frac{1}{A} + \frac{1}{A}B\frac{1}{A}B\frac{1}{A} + \dots$$

we can write Eq.(8) as

$$S_{eff}^{(1)}[A] = -\frac{e^2}{2} \int d^4x \int \frac{d^4p}{(2\pi)^4} \times \text{tr} \left[\frac{1}{p-m} i\partial_\mu A \frac{\partial}{\partial p_\mu} \frac{1}{p-m} \gamma_5 b \frac{1}{p-m} A \right. \\ \left. + \frac{1}{p-m} \gamma_5 b \frac{1}{p-m} i\partial_\mu A \frac{\partial}{\partial p_\mu} \frac{1}{p-m} A \right] \quad (10)$$

after use the relation

$$\frac{\partial}{\partial p_\mu} \frac{1}{p-m} = -\frac{1}{p-m} \gamma^\mu \frac{1}{p-m}.$$

As can be seen by power counting the momentum integral has terms which diverge logarithmically, and to regularize the expression, we use the dimensional regularization method. Carrying out the tr of the γ matrices Eq.(10) takes the form

$$S_{eff}^{(1)}[A] = -\frac{e^2}{2} \int d^4x \int \frac{d^4p}{(2\pi)^4} \left[\frac{N}{(p^2 - m^2)^4} \right], \quad (11)$$

where N is given by

$$N = -4i(p^2 - m^2) [\epsilon^{\alpha\beta\mu\sigma} (3m^2 + p^2) - 4\epsilon^{\alpha\beta\mu\nu} p_\nu p^\sigma] \quad (12)$$

$$\times b_\sigma \partial_\mu A_\alpha A_\beta.$$

Lorentz invariance ensures that

$$\int \frac{d^Dq}{(2\pi)^D} q_\mu q_\nu f(q^2) = \frac{g_{\mu\nu}}{D} \int \frac{d^Dq}{(2\pi)^D} q^2 f(q^2) \quad (13)$$

where D is the dimension of the space time. So, N can be written as

$$N = -12m^2i(p^2 - m^2)\epsilon^{\alpha\beta\mu\sigma} b_\sigma \partial_\mu A_\alpha A_\beta. \quad (14)$$

Observe that the terms that contain p^2 and $p_\nu p^\sigma$ in Eq.(12) cancel when we use the Eq.(13) with $D = 4$. In this way, the logarithmical divergent terms in Eq.(11) disappear, so that we can rewrite the effective action as

$$S_{eff}^{(1)}[A] = \left[6im^2 e^2 \int \frac{d^4p}{(2\pi)^4} \left(\frac{1}{(p^2 - m^2)^3} \right) \right] \quad (15)$$

$$\times \epsilon^{\alpha\beta\mu\sigma} b_\sigma \int d^4x \partial_\mu A_\alpha A_\beta,$$

Note that Eq.(15) is finite by power counting.

From now on, we are interested in the induced Chern-Simons term taking temperature into account.(For a review see [10].) To do this, it is convenient to work in Euclidean space then Eq.(15) can be written as

$$S_{eff}^{(1)}[A] = \left[6m^2 e^2 \int \frac{d^4p_E}{(2\pi)^4} \left(\frac{1}{(p_E^2 + m^2)^3} \right) \right] \quad (16)$$

$$\times \epsilon^{\alpha\beta\mu\sigma} b_\sigma \int (-i)d^4x_E \partial_\mu A_\alpha A_\beta$$

where for Euclidean momentum p_μ , we define the real variable p_4 by $p_0 = ip_4$ Thus, $p^2 = -p_E^2$, and $p_E^2 = \mathbf{p}^2 + p_4^2$, the standard Euclidean square of a vector. Also, we have considered $d^4p = id^4p_E$, $x_0 = -ix_4$, and $d^4x = -id^4x_E$.

If we assume that the system is in thermal equilibrium with a reservoir at temperature β^{-1} we may use the Matsubara formalism. In this case we have to perform the replacements $w \rightarrow w_n = (n + 1/2)2\pi/\beta$ and $(1/2\pi) \int dp_E^0 = 1/\beta \sum_n$. Then Eq.(16) takes the form

$$S_{eff}^{(1)}[A] = -i6e^2 f(m^2, \beta) \epsilon^{\alpha\beta\mu\sigma} b_\sigma \int d^4x_E \partial_\mu A_\alpha A_\beta \quad (17)$$

where $f(m^2, \beta)$ is given by

$$f(m^2, \beta) = \frac{m^2}{\beta} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \sum_{n=-\infty}^{\infty} \left(\frac{1}{(\mathbf{p}^2 + w_n^2 + m^2)^3} \right). \quad (18)$$

First of all, we performing the sum operation in Eq(18). As result, we obtain the polynomial expression of third order in $\tanh(x)$. The follow in step is substitute $\tanh(x) \rightarrow 1 - 2\mathcal{F}_D$ and \mathcal{F}_D is given by

$$\mathcal{F}_D = \frac{1}{\exp(\beta\sqrt{\mathbf{p}^2 + m^2}) + 1}. \quad (19)$$

The function $f(m^2, \beta)$ takes the form

$$f(m^2, \beta) = \int \frac{d^3\mathbf{p}}{(2\pi)^3} [\mathcal{A}(\mathbf{p}^2, m^2, \beta) \quad (20)$$

$$+ \mathcal{B}(\mathbf{p}^2, m^2, \beta)\beta + \mathcal{C}(\mathbf{p}^2, m^2, \beta)\beta^2]$$

The terms $\mathcal{A}(\mathbf{p}^2, m^2, \beta)$, $\mathcal{B}(\mathbf{p}^2, m^2, \beta)$ and $\mathcal{C}(\mathbf{p}^2, m^2, \beta)$ are

$$\mathcal{A}(\mathbf{p}^2, m^2, \beta) = \frac{3}{16} \frac{m^2}{(p^2 + m^2)^{5/2}} [1 - 2\mathcal{F}_D], \quad (21)$$

$$\mathcal{B}(\mathbf{p}^2, m^2, \beta) = -\frac{3}{8} \frac{m^2}{(p^2 + m^2)^2} \mathcal{F}_D [1 - \mathcal{F}_D], \quad (22)$$

and

$$\mathcal{C}(\mathbf{p}^2, m^2, \beta) = -\frac{1}{8} \frac{m^2}{(p^2 + m^2)^{3/2}} \mathcal{F}_D [1 - 3\mathcal{F}_D + 2\mathcal{F}_D] \quad (23)$$

observe that at zero temperature the term $\mathcal{F}_D \rightarrow 0$ and we have

$$\mathcal{A}(\mathbf{p}^2, m^2, \beta \rightarrow \infty) \rightarrow \frac{3}{16} \frac{m^2}{(p^2 + m^2)^{5/2}},$$

while the other terms $\mathcal{B}(\mathbf{p}^2, m^2, \beta \rightarrow \infty) \rightarrow 0$ and $\mathcal{C}(\mathbf{p}^2, m^2, \beta \rightarrow \infty) \rightarrow 0$. So, at zero temperature we get

$$f(m^2, \beta) = \frac{3}{16} \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{m^2}{(p^2 + m^2)^{5/2}}. \quad (24)$$

To evaluate the momentum integration we use the relation

$$\int \frac{d^D p}{(2\pi)^D} \frac{1}{(p^2 + a^2)^s} = \frac{1}{(2\pi)^D} \frac{\pi^{D/2}}{\Gamma(s)} \Gamma(s - D/2) \quad (25)$$

$$\times \frac{1}{(a^2)^{(s-D/2)}},$$

so that the Eq.(24) becomes

$$f(m^2, \beta) = \frac{1}{32\pi^2}. \quad (26)$$

We use this result into Eq.(17) to obtain

$$S_{eff}^{(1)}[A] = \frac{3e^2}{16\pi^2} \epsilon^{\alpha\beta\mu\sigma} b_\sigma \int (-i) d^4 x \partial_\mu A_\alpha A_\beta. \quad (27)$$

This is the Chern-Simons term induced to zero temperature. Observe that this result is finite and is consistent with the computation of the vacuum polarization diagram with fermion propagator $S_F = (\not{p} - m - \gamma_5 \not{b})$ made in the Ref. [1,2].

It is straightforward to verify that in the limit of high temperature, $\beta \rightarrow 0$, the term $\mathcal{F}_D \rightarrow \frac{1}{2}$. Then, $\mathcal{A}(\mathbf{p}^2, m^2, \beta \rightarrow 0) \rightarrow 0$ and the other terms $\mathcal{B}(\mathbf{p}^2, m^2, \beta \rightarrow 0)$ and $\mathcal{C}(\mathbf{p}^2, m^2, \beta \rightarrow 0)$ are different of zero and independent of β . On the other hand, as we can see in Eq.(20) these terms are multiplied by β and β^2 respectively in the numerator thus $f(m^2, \beta \rightarrow 0)$ is zero.

Let us summarize our results: The coefficient of the induced Chern-Simons term behaves monotonically descendent with $\beta(1/T)$, such that when $\beta \rightarrow \infty$ ($T \rightarrow 0$) we reproduce the same result obtained by Jackiw and Kostelecky [1]. In the limit of $\beta \rightarrow 0$ ($T \rightarrow \infty$) the coefficient goes to zero. Therefore, Lorentz and CPT symmetries are restored only when the temperature goes to infinity.

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- [5] S. Carroll, G. Field, and R. Jackiw, Phys. Rev. D **41**, 1231 (1990).
- [6] D. Colladay and V. A. Kostelecky, Phys. Rev. D **55**, 6760 (1997).
- [7] S. Coleman and S. L. Glashow, Phys. Lett. **405B**, 249, (1997); Phys. Rev. D **59**, 116008 (1998).
- [8] M. Goldhaber and V. Trimble, J. Astrophys. Astr. **17**, 17 (1996); S. Carroll and G. Field, Phys. Rev. Lett., **79**, 2394 (1997).
- [9] K. S. Babu, Ashok Das, and Prasanta Panigrahi Phys. Rev. D **36** 3725 (1987); C. M. Fraser, Z. Phys. C **28** 101 (1985); V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Yad. Fiz. **39** 124 (1984) [Sov. J. Nucl. Phys. **39**, 77 (1984)].
- [10] L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974)

- [1] R. Jackiw and V. A. Kostelecky. Phys. Rev. Lett. **82**, 3572 (1999).
- [2] M. Pérez-Victoria, Phys. Rev. Lett, **83**, 2518. (1999).
- [3] D. Colladay and V. A. Kostelecky, Phys. Rev. D **58** 116002 (1998).
- [4] J. M. Chung Phys. Rev. D **60** 12790 (1999); J. M. Chung, Phys Lett B **461** 138 (1999) ; J. M. Chung and P. Oh, Phys Rev D **60** 067702 (1999).